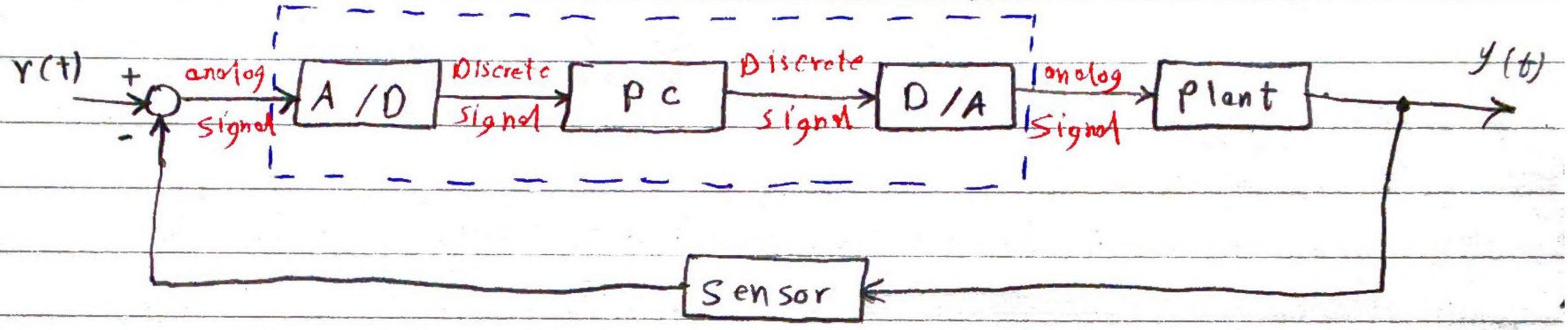
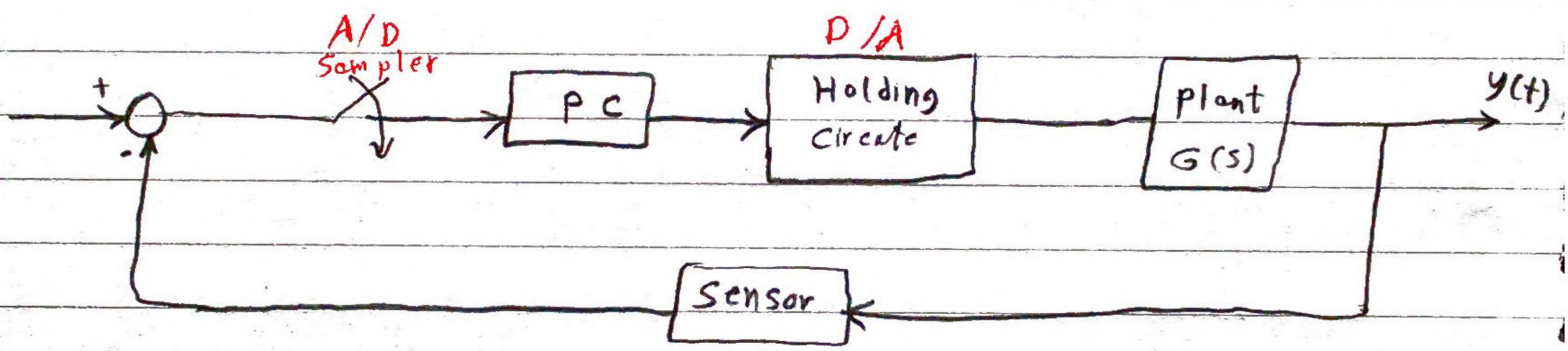
Lec 1

Revision on Z.T.

Digital system:



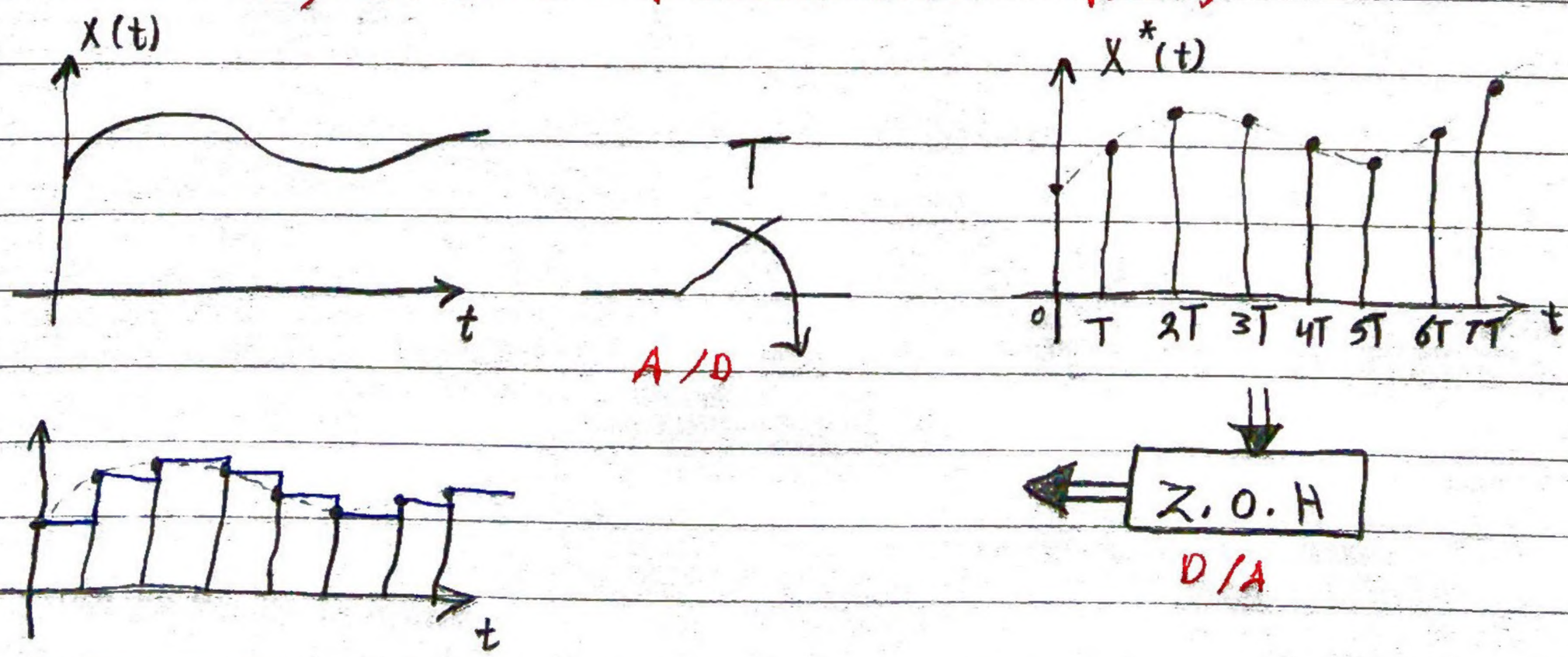


any Digital System Contains:

① A/D

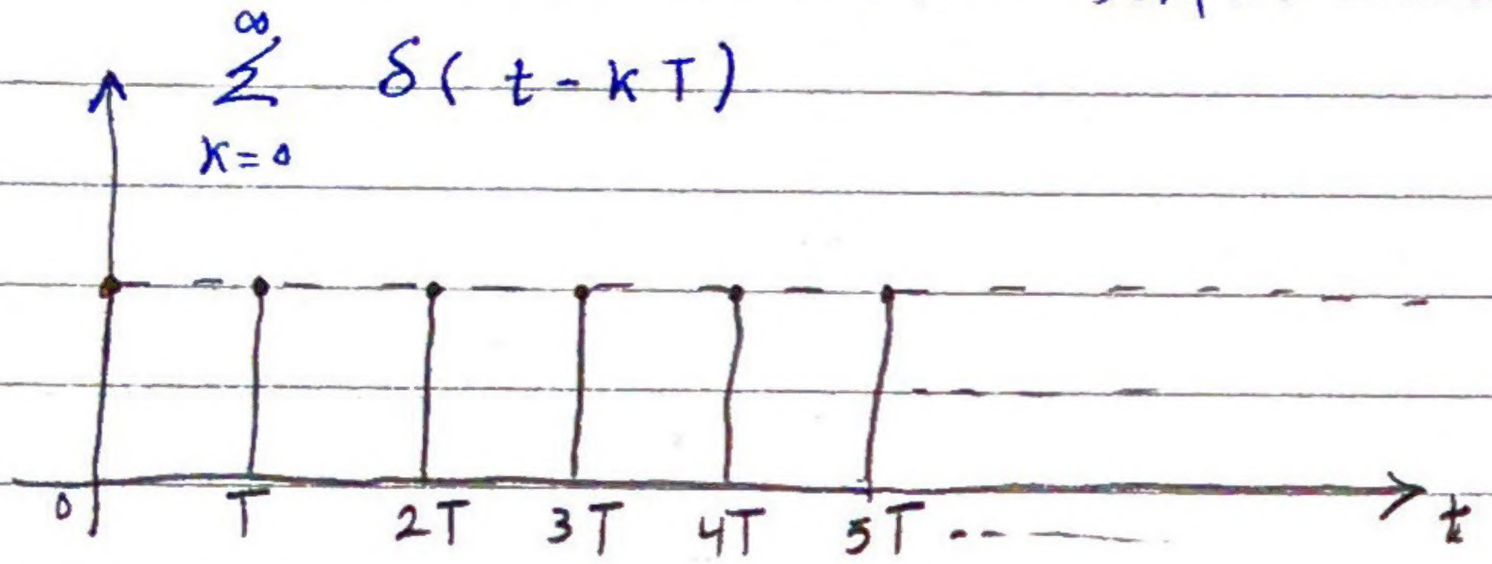
② D/A Converter ≡ holding Circuite

- → Z.O.H (Zero order hold) ✓
- → F.O.H (First order hold)
- → S.O.H (Second order hold)



Can be smothed using L.P.F

The Sampler can be viewed



$$x^*(t) = \sum_{k=0}^{\infty} x(t)\,\delta(t-kT) = x(0) + x(T) + x(2T) + \cdots$$

k → Sample number

T → Sample period

$x^*(t)$ → output of the Sampler

$$x^*(t) = \sum_{k=0}^{\infty} x(t)\,\delta(t-kT) = \sum_{k=0}^{\infty} x(kT)\,\delta(t-kT)$$

→ Laplace Transform

$$X^*(s) = \sum_{k=0}^{\infty} x(kT)\,e^{-kTs}$$

→ Z. Trans Form $\quad z = e^{Ts}$

$$X(z) = \sum_{k=0}^{\infty} x(kT)\,z^{-k}$$

$$x(t) \xrightarrow{Z.T.} X(z) = \sum_{k=0}^{\infty} x(kT)\,z^{-k}$$

Ex: $x(t) = u(t) = 1$

$$X(z) = \sum_{k=0}^{\infty} z^{-k} = 1 + z^{-1} + z^{-2} + z^{-3} + \cdots$$

$$= \frac{1}{1 - z^{-1}} = \frac{z}{z - 1}$$

Ex: $x(t) = e^{at}$

$$X(z) = \sum_{k=0}^{\infty} e^{akT} z^{-k}$$

$$= 1 + e^{aT} z^{-1} + e^{2aT} z^{-2} + \cdots$$

$$= \frac{1}{1 - e^{aT} z^{-1}} = \frac{z}{z - e^{aT}}$$

Ex: $x(t) = \delta(t)$

$$X(z) = \sum_{k=0}^{\infty} \delta(kT) z^{-k} = 1 + 0 + 0 + \cdots = 1$$

Ex: $x(t) = a^t$

$$X(z) = \sum_{k=0}^{\infty} a^{kT} z^{-k}$$

$$= 1 + a^{T} z^{-1} + a^{2T} z^{-2} + \cdots$$

$$= \frac{1}{1 - a^{T} z^{-1}} = \frac{z}{z - a^{T}}$$

Ex: $x(t) = t$

$$X(z) = \sum_{k=0}^{\infty} kT\, z^{-k}$$

$$= 0 + T z^{-1} + 2T z^{-2} + \cdots$$

$$z X(z) = T + 2T z^{-1} + 3T z^{-2} + \cdots$$

$$z X(z) - X(z) = T + T z^{-1} + T z^{-2} + \cdots$$

$$X(z)(z-1) = \frac{T}{1 - z^{-1}} = \frac{Tz}{z-1}$$

$$X(z) = \frac{Tz}{(z-1)^2}$$

Ex: $x(t) = \sin(wt) = \frac{e^{jwt} - e^{-jwt}}{2j}$

$$X(z) = \sum_{k=0}^{\infty} \sin(wkT)\, z^{-k}$$

$$= \sum_{k=0}^{\infty} \frac{1}{2j}\left[e^{jwkT} z^{-k} - e^{-jwkT} z^{-k}\right]$$

$$= \frac{1}{2j}\left[\left(1 + e^{jwT} z^{-1} + e^{2jwT} z^{-2} + \cdots\right) - \left(1 + e^{-jwT} z^{-1} + e^{-2jwT} z^{-2} + \cdots\right)\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1 - e^{jwT} z^{-1}} - \frac{1}{1 - e^{-jwT} z^{-1}}\right]$$

$$= \frac{1}{2j}\,\frac{1 - e^{-jwT} z^{-1} - 1 + e^{jwT} z^{-1}}{1 - e^{jwT} z^{-1} - e^{-jwT} z^{-1} + z^{-2}}$$

$$= \frac{1}{2j}\,\frac{z\left(e^{jwT} - e^{-jwT}\right)}{z^2 - z\left(e^{jwT} + e^{-jwT}\right) + 1}$$

$$X(z) = \frac{z\left(\frac{e^{jwT} - e^{-jwT}}{2j}\right)}{z^2 - 2z\left(\frac{e^{jwT} + e^{-jwT}}{2}\right) + 1}$$

$$= \frac{z \sin(wT)}{z^2 - 2z\cos(wT) + 1}$$

Ex: $x(t) = \cos wt$

$$X(z) = \frac{z(z - \cos(wT))}{z^2 - 2z\cos(wT) + 1}$$

Properties of Z.T. :

1. $f_1(t) \pm f_2(t) \xrightarrow{Z.T} F_1(z) \pm F_2(z)$
2. $a\,f(t) \xrightarrow{Z.T} a\,f(z)$
3. $e^{\pm at} f(t) \xrightarrow{Z.T} f(z\,e^{\mp aT})$ (عكس الإشارة)
4. $a^t f(t) \xrightarrow{Z.T} f\left(\frac{z}{a^T}\right)$
5. $t\,f(t) \xrightarrow{Z.T} -Tz\frac{df(z)}{dz}$
6. initial value

$$f(0) = \lim_{t\to 0} f(t) = \lim_{z\to\infty} F(z)$$

7. Final value

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{z\to 1} (z-1)\,F(z)$$

Ex: $y(k+2) + 3y(k+1) + 2y(k) = \delta(k)$

if $y(0) = 0$, $y(1) = -1$, Solve for $y(k)$

→ using Z.T.

$$z^2 y(z) - z^2 y(0) - z y(1) + 3[z Y(z) - z y(0)]$$

$$+ 2 Y(z) = 1$$

$$z^2 Y(z) + z + 3z Y(z) + 2 Y(z) = 1$$

$$Y(z)[z^2 + 3z + 2] = 1 - z$$

$$Y(z) = \frac{1-z}{z^2+3z+2} = \frac{1-z}{(z+1)(z+2)}$$

$$= \frac{A}{z+1} + \frac{B}{z+2}$$

$A = 2$

$B = -3$

$$Y(z) = \frac{2}{z+1} - \frac{3}{z+2} = 2z^{-1}\frac{z}{z+1} - 3z^{-1}\frac{z}{z+2}$$

→ Inverse Z.T.

$$y(k) = 2(-1)^{k-1} u(k-1) - 3(-2)^{k-1} u(k-1)$$

Ex : $F(z) = \dfrac{z(z+1)}{(z+2)(z+4)}$

$$= z\left[\frac{A}{z+2} + \frac{B}{z+4}\right]$$

$A = \frac{-1}{2}$ $\quad B = +\frac{3}{2} = 1.5$

$$= \frac{-1}{2}\frac{z}{z+2} + 1.5\frac{z}{z+4}$$

$$F(k) = \frac{-1}{2}(-2)^k u(k) + 1.5(-4)^k u(k)$$

Report. $F(z) = \dfrac{z(z+1)}{(z+2)(z+4)}$ For $T = 0.5$ sec